A Revision of Cascade Synthesis Theory Covering Cross-Coupled Filters

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***Abstract*—The classical zero-shifting technique is generalized to cover extraction of complex transmission zeros (TZs) in the form of fourth-order *LC* sections whereby the $j\omega$- and α-axis TZs appear as special cases. Using this approach, bandpass filters can be synthesized in direct coupled resonator forms by pole placement instead of designing them through low-pass prototypes. By using circuit transformations, the resulting direct coupled resonator filter circuits can then be transformed into a variety of cross-coupled forms like a fully cross-coupled form or cascaded N-tuplet form. It is shown that one or more finite $j\omega$-axis, α-axis, or complex TZs can be extracted as direct coupled resonator circuit blocks, which can be converted into cross-coupled triplets, quadruplets, or other N-tuplets of resonators. In particular, it is shown that a cascaded quadruplet section can be used to realize a complex TZ quadruplet $s_i = \pm\sigma_i \pm j\omega_i$, as well as two pairs of $j\omega$-axis TZs, $s_i = \pm j\omega_i$, and $s_k = \pm j\omega_k$.**

***Index Terms*—Cross-coupled, filters, linear phase, synthesis.**

I. Introduction

CASCADE synthesis for direct design of filters by placement of transmission zeros (TZs) is a well-established technique. Especially for bandpass filters, compared to design techniques involving frequency transformations from low-pass prototypes, this technique has advantages of involving no approximations and having maximum flexibility for shaping both the amplitude and phase response of filters by adjusting locations of TZs [1]–[3]. In this paper, the cascade synthesis technique will be generalized to cover the direct bandpass design of cross-coupled resonator filters by placement of TZs. In the new approach, the cross-coupled filters will be treated as extensions of direct-coupled resonator filters, which are shown in Fig. 1(a) in a shunt LC resonator form with simple L- or C-type coupling elements. In the simplified schematics, the shunt resonators will be shown as circles and coupling elements as heavy lines. They can be designed through cascade synthesis by placement of TZs only at $s = 0$ and $s = \infty$. However, such structures cannot meet the extreme selectivity, flat delay, and miniaturization requirements of modern applications. Restrictions on both

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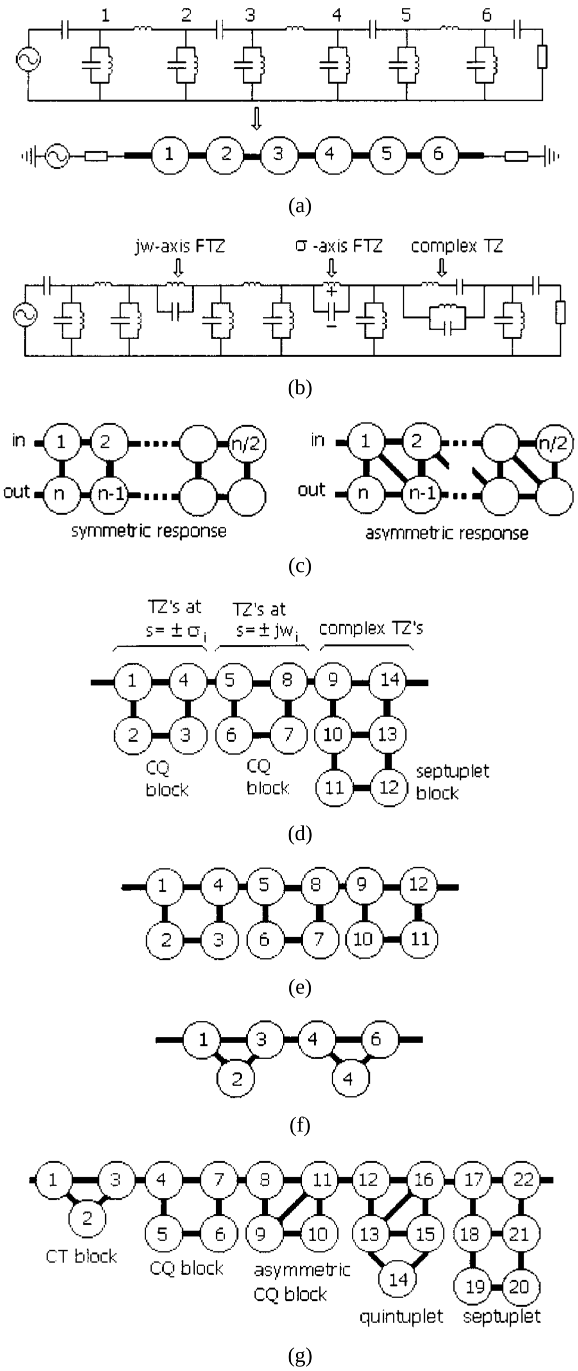


Fig. 1. Some direct- and cross-coupled resonator filter structures. (a) Direct-coupled resonator filter without FTZs. (b) Direct-coupled resonator filter with FTZs. (c) Fully cross-coupled filters. (d) Pfitzenmaier's low-pass prototype approach. (e) CQ filter. (f) CT filter. (g) Mixed cascaded N-tuplet filter.

the amplitude and phase responses necessitate placement of finite $j\omega$-axis and/or complex TZs, which lead to impractical coupling elements when synthesized in cascade form, as shown in Fig. 1(b). All these problems are resolved by introducing cross couplings between nonadjacent resonators, as described in the pioneering works of Rhodes [5], Atia and Williams [6], [7], Pfitzenmaier [8], [9], Bell [10], Cameron [11], and Rhodes and Cameron [12]. In these papers, it was demonstrated that the fully cross-coupled (FCC) structure shown in Fig. 1(c) can readily satisfy linear phase and/or high-selectivity requirements with the diagonal cross couplings arising when the amplitude response is asymmetric. The succeeding research efforts are concentrated on the development of special techniques and tools for synthesis of such FCC filters [13]–[21]. Since each of the cross couplings contribute to all of the finite transmission zeros (FTZs), their tuning is problematic. Therefore, as a transition between direct-coupled filters and FCC filters, Pfitzenmaier [8] introduced low-pass prototypes leading to structures formed by cascading four-resonator [cascaded quadruplet (CQ)] blocks for realizing either σ-axis ($s_i = \pm\sigma_i$) or $j\omega$-axis ($s_i = \pm j\omega_i$) TZs and six-resonator blocks for realizing either a complex conjugate quadruplet of TZs $s_j = \pm\sigma_j \pm j\omega_0$ or two $j\omega$-axis TZs $s_j = \pm j\omega_j$ and $s_k = \pm j\omega_k$, as described in Fig. 1(d). Due to the close correspondence between cross-coupled blocks and the TZs, tuning of TZs is easier in these structures compared to the FCC filters. Levy [13] then introduced CQ filters [see Fig. 1(e)]. Both Pfitzenmaier's and Levy's approaches lead to symmetric amplitude response, as their approaches are based on low-pass prototypes. Next, coupled triplets are introduced for the realization of single $j\omega$-axis TZs for asymmetric amplitude response [14], [17], [19]–[21], leading to cascaded triplet (CT) filters [see Fig. 1(f)]. Eventually these contributions led to the conclusion that N-resonator blocks can be cascaded, as shown in Fig. 1(g), for shaping both amplitude and phase responses of the filters, as alternatives to the FCC filters. In this paper, the cascaded block approach of Pfitzenmaier will be generalized for direct synthesis of bandpass filters by placement of TZs instead of using low-pass prototypes. This approach avoids limitations due to the LP-to-BP mapping functions, thus leading to more general structures, which have extra flexibility for response shaping, like complex TZ CQ sections without diagonal cross-couplings for asymmetric amplitude responses. The approach is based on the classical cascade synthesis technique followed by circuit transformations for conversion of some proper sections of the resulting structure into CT, CQ, or any other N-tuplet blocks.

Along this line, extraction of complex TZs formed a barrier because, in the classical approach, they are extracted as Darlington-D sections, which are not suitable for easy handling [4]. This problem is resolved by developing a novel approach in which complex TZs are extracted as fourth-order sections, which can readily be converted into CQ sections. This technique is described in Section II, together with a revision of formulation of transfer functions with TZ placement in the transformed frequency domain. In Section III, conversion of properly extracted circuit sections into different cross-coupled topologies will be described qualitatively. Design examples will be given in Sections IV and V.

II. Formulation of Transfer Function and Element Extraction

Transducer power gain of a passive lossless reciprocal two-port can be expressed in the form

$$S_{21}(s)S_{21}(-s) = \frac{1}{1+K(s)K(-s)} \tag{1}$$

where $K(s)$ is termed as the characteristic function of the filter defined as

$$K(s)K(-s) = \varepsilon^2\frac{f(s)f(-s)}{p(s)p(-s)} = \pm\varepsilon^2\frac{f^2(s)}{p^2(s)}. \tag{2}$$

ε is termed as the passband ripple factor. $f(s)$ and $p(s)$ are the even or odd polynomials with real coefficients, which are related to each other through the Feldtkeller equation

$$e(s)e(-s) = f(s)f(-s) + p(s)p(-s) \tag{3}$$

with $e(s)$ being a strictly Hurwitz polynomial. For typical equiripple or maximally flat bandpass filters, $p(s)$ and $f(s)$ are of the form

$$\begin{aligned} p(s) &= s^{n0}\prod\left(s^2+\omega_i^2\right)\prod\left[(s+\sigma_k)^2+\omega_k^2\right] \\ &\quad\times\left[(s-\sigma_k)^2+\omega_k^2\right] \\ f(s) &= \prod\left(s^2+\omega_r^2\right) \end{aligned} \tag{4}$$

and, hence,

$$\begin{aligned} &K(s)K(-s) \\ &\quad= \pm\varepsilon^2\frac{f^2(s)}{p^2(s)} \\ &\quad= \frac{\prod\left(s^2+\omega_r^2\right)^2}{s^{2n0}\prod\left(s^2+\omega_i^2\right)^2\prod\left[(s+\sigma_k)^2+\omega_k^2\right]^2\left[(s-\sigma_k)^2+\omega_k^2\right]^2} \end{aligned} \tag{5}$$

where n_0 is the number of TZs at $s = 0, \omega_i$'s and ω_r's are $s = j\omega$-axis TZs and reflection zeros (RZs), respectively. The third factor in $p(s)$ is formed by the complex conjugate quadruplets of TZs $s_k = \pm\sigma_k \pm j\omega_k$. The degree difference between $e(s)$ and $p(s)$ sets the number of TZs at $s = \infty$. In typical synthesis problems, one can force $K(s)K(-s)$ to have equiripple or maximally flat passband amplitude response by specifying the TZs only, in which case, RZs are automatically set. That is, $f(s)$ can be recognized as the numerator of $K(s)K(-s)$. Knowing $p(s)$ and $f(s)$, the polynomial $e(s)e(-s)$ can be found from the Feldtkeller equation from which $e(s)$ is formed using the left half-plane roots of $e(s)e(-s) = 0$. After $e(s), f(s)$, and $p(s)$ are obtained, one can form any one of the two-port parameters of the circuit, like S, Z, Y, or $ABCD$ for element extraction. However, due to severe ill conditioning, finding the roots of $f(s)f(-s) = 0$ and $e(s)e(-s) = 0$ are problematic. Such accuracy problems are reduced significantly if the whole synthesis is carried out in the transformed frequency domain [2], [3], as summarized below.

A. Formulation of Impedance Functions in Transformed Domain

The following frequency transformation maps the passband of bandpass filters on the $s = j\omega$-axis onto the whole imaginary axis of the transformed domain [2], [3]:

$$z^2 = \frac{s^2 + \omega_{p2}^2}{s_n^2 + \omega_{p1}^2} \quad \mathrm{Re}(z) > 0, \qquad \text{for BPF} \tag{6}$$

where ω_{p1} and ω_{p2} are the passband edge frequencies. This transformation separates the zeros clustered in and near the passband, easing numerical accuracy problems. Under this transformation, a complex TZ quadruplet $s_i = \pm\sigma_i \pm j\omega_i$ is mapped onto the $z = x + jy$ plane as $Z_i = X_i \pm jY_i$ where

$$\begin{aligned} X_i &= \sqrt{X_o^2 + Y_o^2}\cos\frac{\theta}{2} \\ Y_i &= \sqrt{X_o^2 + Y_o^2}\sin\frac{\theta}{2} \\ X_0 &= \frac{\left(1+\sigma_i^2-\omega_i^2\right)\left(a^2+\sigma_i^2-\omega_i^2\right)+4\omega_{i'}^2\sigma_i^2}{\left(a^2+\sigma_i^2-\omega_i^2\right)^2+4\omega_i^2\sigma_i^2} \\ Y_0 &= \frac{2\omega_i\sigma_i(a^2-1)}{\left(a^2+\sigma_i^2-\omega_i^2\right)^2+4\omega_i^2\sigma_i^2} \\ \theta &= \tan^{-1}\frac{Y_o}{X_o}. \end{aligned} \tag{7}$$

The mapped forms of $s = 0, \infty, j\omega$-axis and σ-axis TZs can be found as special cases of (7). Denoting the transformed versions of the polynomials $f(s)$ and $p(s)$ by $F(z^2)$ and $P(z^2)$, respectively, the transformed version of $K(s)K(-s)$ can be written as

$$K\left(z^2\right)\overline{K}\left(z^2\right) = \frac{F\left(z^2\right)\overline{F}\left(z^2\right)}{P\left(z^2\right)\overline{P}\left(z^2\right)}. \tag{8}$$

Consider the polynomial $V(z)$ defined in terms of the transformed versions of the TZs Z_i as

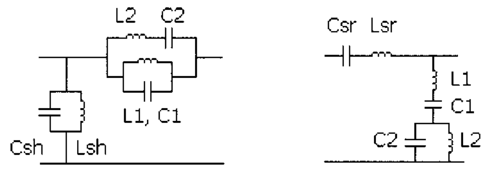
$$V(z) = \prod_{i=1}^{n1}(Z_i + z)\prod_{i=1}^{n2/2}\left(z^2 + 2X_iz + X_i^2 + Y_i^2\right) \tag{9}$$

where the first term is due to $n_1 j\omega$-axis TZs, including those at $s = 0$ and $s = \infty$, and the second term is due to the complex TZs with n_2 being the total degree. Since complex TZs are placed as quadruples, n_2 and $n_2/2$ are always even. Consider the following function formed by $V(z)$ and $V(-z)$:

$$K\left(z^2\right)\overline{K}\left(z^2\right) = \varepsilon^2\frac{1}{4}\left(1+\frac{V(z)}{V(-z)}\right)\left(1+\frac{V(-z)}{V(z)}\right). \tag{10}$$

Equiripple property of this function can readily be proven by using complex algebra. Let

$$\begin{aligned} \gamma_i &= \cosh^{-1}\left(\frac{Z_i}{\sqrt{Z_i^2 - z^2}}\right) \\ \chi_i &= \cosh^{-1}\left(\frac{|z^2 + |Z_i|^2|}{\sqrt{(z^2 + |Z_i|^2)^2 - 4X_i^2z^2}}\right) \\ \frac{(Z_i + z)}{(Z_i - z)} &= e^{2\gamma_i}, \frac{z^2 + 2X_iz + X_i^2 + Y_i^2}{z^2 - 2X_iz + X_i^2 + Y_i^2} = e^{2\chi_i}. \end{aligned} \tag{11}$$



(a) (b)

Fig. 2. Complex TZ sections.

Using these polar forms, $V(z)/V(-z)$ can be written as

$$\begin{aligned} \frac{V(z)}{V(-z)} &= \prod_{i=1}^{n1}\frac{(Z_i+z)}{(Z_i-z)}\prod_{i=1}^{n2}\frac{z^2+2X_iz+X_i^2+Y_i^2}{z^2-2X_iz+X_i^2+Y_i^2} \\ &= \prod_{i=1}^{n1}e^{2\gamma_i}\prod_{i=1}^{n2}e^{2\chi_i} \\ &= e^{\sum 2\gamma_i}e^{\sum 2\chi_i} \\ &= e^{2\gamma}e^{2\chi} \end{aligned} \tag{12}$$

where γ and χ are defined as

$$\gamma = \sum\gamma_i \text{ and } \chi = \sum\chi_i. \tag{13}$$

Using (12) in (10), we get the form

$$K(z^2)\overline{K}(z^2) = \varepsilon^2\cosh^2\left[\begin{array}{c}\sum_{i=1}^{n1}\cosh^{-1}\left(\frac{Z_i}{\sqrt{Z_i^2-z^2}}\right) \\ +\sum_{i=1}^{n2}\cosh^{-1}\left(\frac{|z^2+|Z_i|^2|}{\sqrt{(z^2+|Z_i|^2)^2-4X_i^2z^2}}\right)\end{array}\right] \tag{14}$$

which is equiripple in passband and have the specified TZs. After forming $K(z^2)\overline{K}(z)^2$, the polynomials $F(z^2), P(z^2)$ and $E(z^2)$ can be determined using the classical approaches [3].

B. Element-Extraction Procedure

Element extractions can be carried out from z_{11}- and y_{11}-parameters of the two-port

$$y_{11}R_1 = \frac{\mathrm{Odd}\left(E(z^2)\right) + \mathrm{Odd}\left(F(z^2)\right)}{\mathrm{Even}\left(E(z^2)\right) \mp \mathrm{Even}\left(F(z^2)\right)} \tag{15}$$

$$\frac{z_{11}}{R_1} = \frac{\mathrm{Odd}\left(E(z^2)\right) \pm \mathrm{Odd}\left(F(z^2)\right)}{\mathrm{Even}\left(E(z^2)\right) + \mathrm{Even}\left(F(z^2)\right)} \tag{16}$$

where the upper and lower signs refer to symmetric and antimetric circuits, respectively. In cascade synthesis, each extracted element or circuit section realizes a certain TZ. A finite TZ is realized by shifting a zero either from $s = 0$ or from $s = \infty$. Complex TZs are traditionally extracted as Darlington-D sections, which are rather complicated structures for realization [4]. One novelty of this paper is that complex TZs are extracted either as a series or shunt fourth-order section having some negative elements, as shown in Fig. 2(a), obtained from (15), or Fig. 2(b), obtained from (16). This is possible by shifting zeros at both $s = 0$ and $s = \infty$ simultaneously to create the complex TZ at the desired location. That is, the impedance or the admittance functions shown in (15) or (16) must have zeros at both $s = 0$ and $s = \infty$.

The procedure can be summarized as follows. Denoting the complex TZ by $s_0 = \pm\sigma_0 \pm j\omega_0$, which corresponds to $Z_0 = \pm X_0 \pm jY_0$ in the z-domain, extraction of the shunt elements $L_{\rm sh}$ and $C_{\rm sh}$ of Fig. 2(a) leads to the relation

$$Y_1(z)|_{z=Z_0} = Y_{LC}(z)|_{z=Z_0} + Y_2(z)|_{z=Z_0} \tag{17}$$

where $Y_1(z)$ is the original input admittance and $Y_2(z)$ is the admittance of the remaining circuit after removal of the pair $L_{\rm sh}$–$C_{\rm sh}$. The remaining impedance $Z_2(z) = 1/Y_2(z)$ will have a pole at $Z_0 = \pm X_0 \pm jY_0$, which can be extracted as a series arm fourth-order section whose impedance $Z_q(z)$ can be expressed in terms of four unknowns L_1, C_1, L_2, C_2 in the z-domain as

$$Z_q(z) = \frac{(z^2-1)\sqrt{z^2-1}\sqrt{1-a^2z^2}}{\left((\sigma_0^2-\omega_0^2+a^2)^2+4\omega_0^2\sigma_0^2\right)} \times \frac{\left(\dfrac{1-a^2z^2}{z^2-1}\dfrac{1}{C_1}+\dfrac{1}{L_2C_2C_1}\right)}{(z-Z_{01})(z-Z_{02})(z-Z_{03})(z-Z_{04})}. \tag{18}$$

In order to have the complex TZ at the desired location, the fourth-order section element values should satisfy the following equations:

$$\left(\frac{1}{L_2C_2}+\frac{1}{L_1C_1}\frac{1}{L_2C_1}\right) = -2(\sigma_0^2-\omega_0^2)$$
$$\frac{1}{L_2C_2L_1C_1} = \left(\sigma_0^2+\omega_0^2\right)^2. \tag{19}$$

On the other hand, $Z_2(z)$ and $Z_q(z)$ are related as

$$Z_2(z)\left[(z-Z_{01})(z-Z_{02})(z-Z_{03})(z-Z_{04})\right]|_{z=Z_0} = Z_q(z)\left[(z-Z_{01})(z-Z_{02})(z-Z_{03})(z-Z_{04})\right]|_{z=Z_0}. \tag{20}$$

Solving (17)–(20), the unknowns L_1, L_2, C_1, and C_2 can be found. The shunt fourth-order section of Fig. 2(b) can be extracted in the same way, but by starting from (16). In both fourth-order sections, the elements L_1 and C_1 come out to be negative. The negative element problems will be resolved after conversion into a cross-coupled form.

The same formulation can be used to realize a pair of $j\omega$-axis TZs $s_i = \pm j\omega_i$ and $s_k = \pm j\omega_k$, resulting in the same circuit, but with all positive element values. Single $j\omega$-axis and σ-axis TZs can be extracted through the same formulation, as special cases by placing either $\sigma_i = 0$ or $\omega_i = 0$, leading to the well-known Brune and Darlington-C sections.

III. Conversion Into Cross-Coupled Resonator Forms

Typical coupled resonator filters can be realized by extracting the TZs as cascaded blocks as follows.

1) TZs at $s = 0$ and $s = \infty$ can be extracted as series and shunt inductors or capacitors, as shown in Fig. 3(a), in groups of $N_{\rm zero} + N_{\rm inf} = \text{Even Integer}$. Fig. 3(a) shows the cases with $N_{\rm zero} = 1, N_{\rm inf} = 3$ and $N_{\rm zero} = 3, N_{\rm inf} = 1$. In the remainder of this paper, the shorthand notation $N(1,3)$ and $N(3,1)$ will be used to denote the TZ pairs $N_{\rm zero} = 1, N_{\rm inf} = 3$ and $N_{\rm zero} = 3, N_{\rm inf} = 1$, respectively, as they will be referred to frequently.

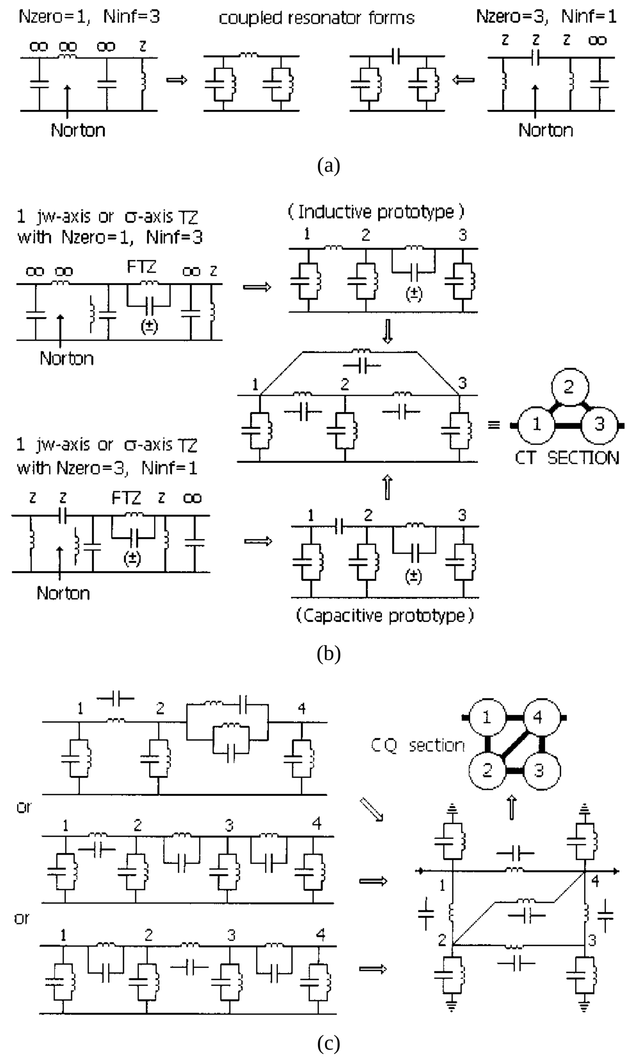


Fig. 3. Suggested approaches for extraction of TZs. (a) At $s = 0, s = \infty$ in groups. (b) Single $j\omega$-axis or σ-axis TZs in CT form (together with $N_{\rm zero} = 1, N_{\rm inf} = 3$ or $N_{\rm zero} = 1$). (c) Realization of a complex TZ quadruplet $s_i = \pm\sigma_i \pm j\omega$ or two $j\omega$-axis TZs $s_k = \pm j\omega_k$ as a CQ section (together with $N_{\rm zero} = 1, N_{\rm inf} = 3$ or $N_{\rm zero} = 3, N_{\rm inf} = 1$).

The coupled resonator form can be obtained by application of the Norton transformation to the series element. The Norton transformer can be used to adjust the element values, such as equating shunt inductors or capacitors.

2) A $j\omega$-axis TZ can be extracted as a Brune section (series arm parallel *LC*), while a σ-axis TZ can be extracted as a Darlington-C section (series arm parallel *LC* with negative capacitor) by zero shifting. However, if cross-coupling is intended, then a $j\omega$-axis or σ-axis TZ can also be extracted as part of a sixth-order circuit block with either $N(1,3)$ or $N(3,1)$, as shown in Fig. 3(b). In these circuits, both $j\omega$-axis and σ-axis TZs appear as a series arm parallel *LC*, with a capacitor of σ-axis TZ being negative. The resulting circuit is then converted into a direct-coupled resonator form by applying Norton transformations to both series elements. The whole circuit is then converted into a CT section by applying row–column operations applied to the admittance matrix of the direct-coupled resonator circuit, as described in

[14] and [19]. Row–column operations target elimination of unwanted couplings, reduction of LC-type coupling into simple L-or C-type coupling, and introduction of the desired coupling, resulting in several distinct solutions. In the resulting CT forms, coupling element types and values depend on both the location of the TZ with respect to the passband and also on $N_{\rm zero}$ and $N_{\rm inf}$.

3) As it's shown in the previous section, a complex TZ quadruplet $s_i = \pm\sigma_i \pm j\omega_i$ can be extracted in the form of a fourth-order section having some negative elements by shifting zeros from both $s = 0$ and $s = \infty$. The negative element problem can be solved only if the complex TZ is extracted as part of a cross-coupled quadruplet. This is described in Fig. 3(c). In this technique, the complex TZ is extracted as part of an eighth-order circuit block with either $N(1,3)$ or $N(3,1)$. The circuit section is then converted into direct-coupled resonator form by Norton transformations and then converted into a CQ section by applying row–column operations on the 4 × 4 admittance matrix of the direct-coupled resonator circuit. As in the CT case, more than one solution is possible [20]. Experiments have shown that a good delay flatness and widest flat delay bandwidth can be obtained if $\sigma_i \approx (\omega_{p2} - \omega_{p1})/2$ and $\omega_i \approx \omega_0$. Here, $\omega_{p1,2}$ are the passband edge frequencies and ω_0 is the geometric center of passband. These σ_i and ω_I values also lead to a low diagonal cross coupling, which can be eliminated by tuning ω_I to a critical frequency about ω_0. This critical frequency and coupling element types (inductive or capacitive) depend on the numbers of TZs at $f = 0$ and $f = \infty$, as well as on the numbers and positions of $j\omega$-axis FTZs. The nature of diagonal coupling (inductive or capacitive) changes above and below this critical frequency. The ability to eliminate diagonal coupling is a feature of direct placement of the complex TZs. This is not possible in the classical LP-to-BP mapping approaches where a CQ section realizing a complex TZ is mapped from the σ-axis TZ of the LP prototype [9]. In such filters, diagonal couplings cannot be eliminated unless the $j\omega$-axis TZs are symmetric. This is because only the σ_i components of the complex TZ can be adjusted by tuning the σ-axis TZ of the LP prototype. This feature may be an advantage of the direct placement of complex TZs described in this paper, which allows tuning of both σ_i and ω_i for both response shaping and elimination of diagonal cross-coupling of the complex CQ sections.

4) CQ sections may also be used to realize two $j\omega$-axis TZs $s_i = \pm j\omega_i$ and $s_k = \pm j\omega k$ simultaneously, as shown in Fig. 3(c). The two $j\omega$-axis TZs may be extracted in different orders in a circuit block with either $N(1,3)$ or $N(3,1)$. The two $j\omega$-axis TZs may then be combined to form a single fourth-order section. The row–column operations can be applied on any one of the three structures to get the CQ form. The diagonal cross-coupling can disappear only by locating the two $j\omega$-axis TZs symmetrically about the passband.

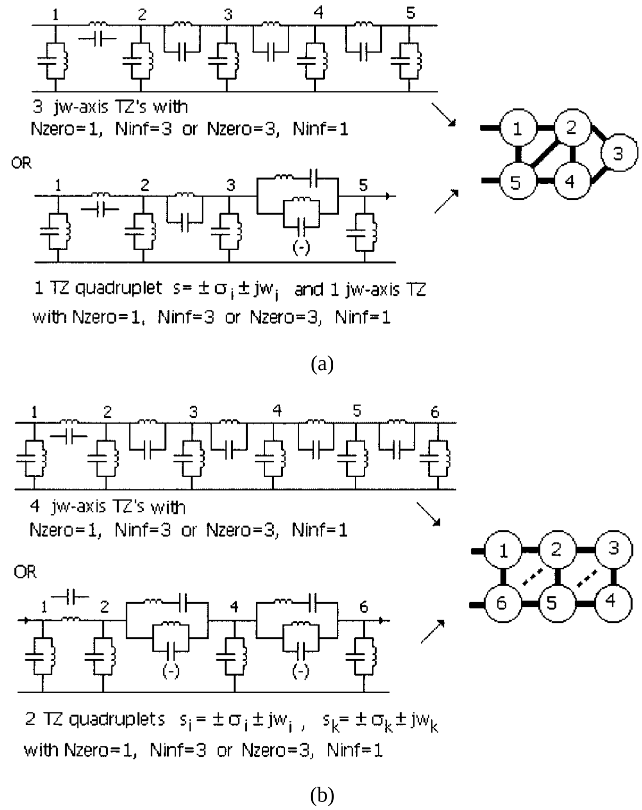


Fig. 4. Five- and six-resonator cross-coupled blocks. (a) Quintuplets. (b) Septuplets.

5) The same approach can also be applied to form higher order cross-coupled modules to realize more than two $j\omega$-axis or complex TZs as a single N-tuplet. Fig. 4(a) shows formation of a five-resonator block called a "quintuplet." It is extracted as a degree-10 circuit section with either $N(1,3)$ or $N(3,1)$. A quintuplet can realize either three $j\omega$-axis FTZs or a complex TZ and a single $j\omega$-axis finite TZ. Fig. 4(b) shows the formation of a six-resonator block called a "septuplet." It is extracted as a degree-12 circuit section with either $N(1,3)$ or $N(3,1)$. A septuplet can realize either four $j\omega$-axis FTZs or two complex TZs.

Conversion from direct-coupled forms to quintuplet or septuplet forms is carried out again by row–column operations applied on admittance matrices. However, matrix algebra gets tedious as the number of resonators gets higher. Therefore, N-tuplets of higher degree may be easier to handle with the approaches described in [15]–[18], which involve both analytical tools and optimization.

In summary, one can form CT, CQ, or mixed CT–CQ–quintuplet–septuplet filters by extracting the relevant circuit blocks in any order and then converting them into N-tuplet forms.

It should be noted that each N-tuplet requires a total of at least four TZs at $s = 0$ and $s = \infty$. Thus, a single N-tuplet (FCC) filter with N_R resonators will have the maximum possible number of FTZs of $2N_R - 4$. All other filters formed by cascading several N-tuplets with the same number of resonators will have fewer FTZs because each N-tuplet section needs $N_{\rm zero}+N_{\rm inf} = 4$. The difference between $N_{\rm zero}+N_{\rm inf}$ of a single and a multiple N-tuplet filter of the same degree reflects itself as limitations on amplitude or phase response as follows.

- When highest skirt selectivity is required, the single N-tuplet version (FCC) is advantageous because it can collect all of its $2N_R - 4$ degrees as $j\omega$-axis FTZs near the band edges at the expense of lower minimum stopband insertion loss. A cascaded N-tuplet filter can locate fewer FTZs there, but minimum stopband loss will be higher.
- Similarly, in FCC filters, all the $2N_R - 4$ degrees can be realized as $(2N_R - 4)/4$ complex TZs to get the widest flat delay bandwidth at the expense of less selectivity while a lesser number of complex TZs can be realized in cascaded N-tuplet filters with the same number of resonators, leading to narrower flat delay bandwidth, but having higher selectivity. Thus, selectivity, flat delay bandwidth, and minimum stopband loss level need to be compromised.

IV. Theoretical Design Example

In this section, an example will be presented involving two CQ sections, one to create two $j\omega$-axis TZs in the upper stopband and the other to create a complex TZ to flatten the delay. Filter specifications are as follows:

passband edges	790–810 MHz;
passband ripple	0.1 dB;
3 TZs	at both $f = 0$ and $f = \infty$;
two finite $j\omega$-axis TZs	at 840 and 860 MHz;
one complex TZ	at $\pm 10 \pm j799.7$ MHz.

The TZs are extracted in the order shown in Fig. 5(a). Fig. 5(b) shows the coupled resonator form after Norton transformations. Using matrix operations, the filter is then converted into CQ form, as given in Fig. 5(c).

The complex TZ is selected in accordance with the constraint $\sigma_i \approx (\omega_{p2} - \omega_{p1})/2$ and $\omega_i \approx \omega_0$ and ω_i is tuned until the diagonal coupling element of the complex CQ section is eliminated. Since the two $j\omega$-axis TZs are on the same side of the passband (unsymmetrical TZs), the diagonal coupling of the CQ section corresponding to these TZs cannot be eliminated. Response of the filter is shown in Fig. 5(d). It is seen that the delay is flattened within 50% of the passband with only one complex TZ quadruplet. Fig. 5(e)–(f) shows two other solutions obtained by extracting the TZs in different orders.

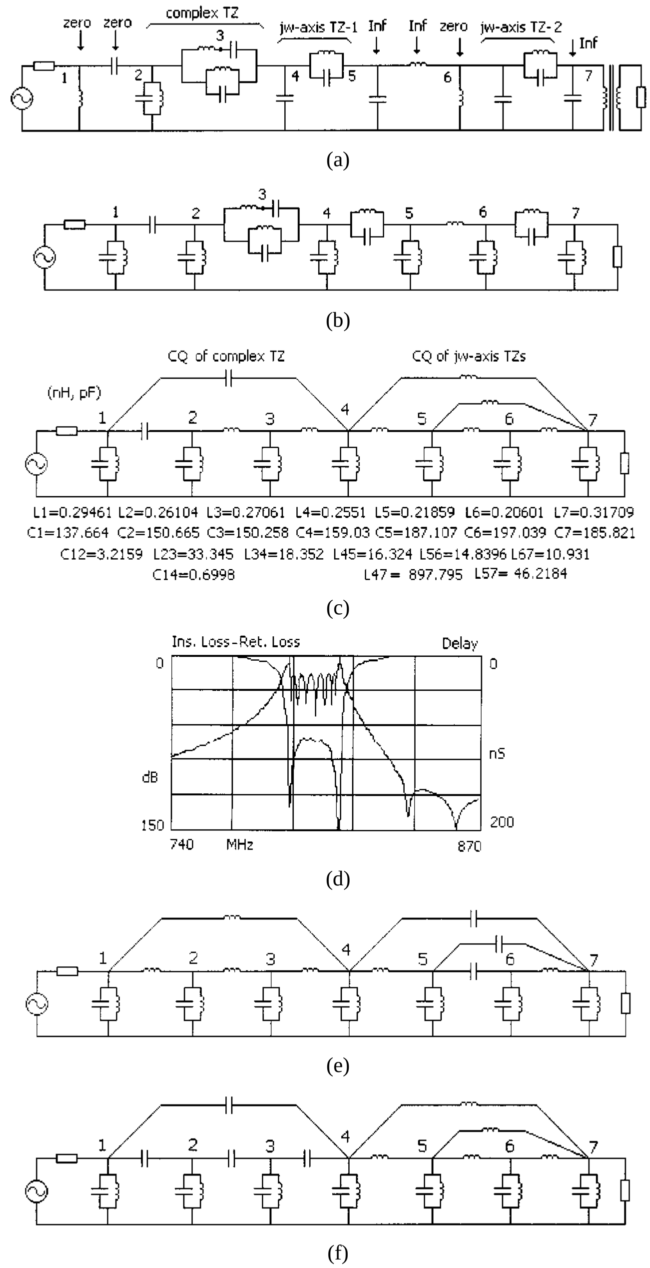


Fig. 5. Development of the CQ filter and other possible solutions.

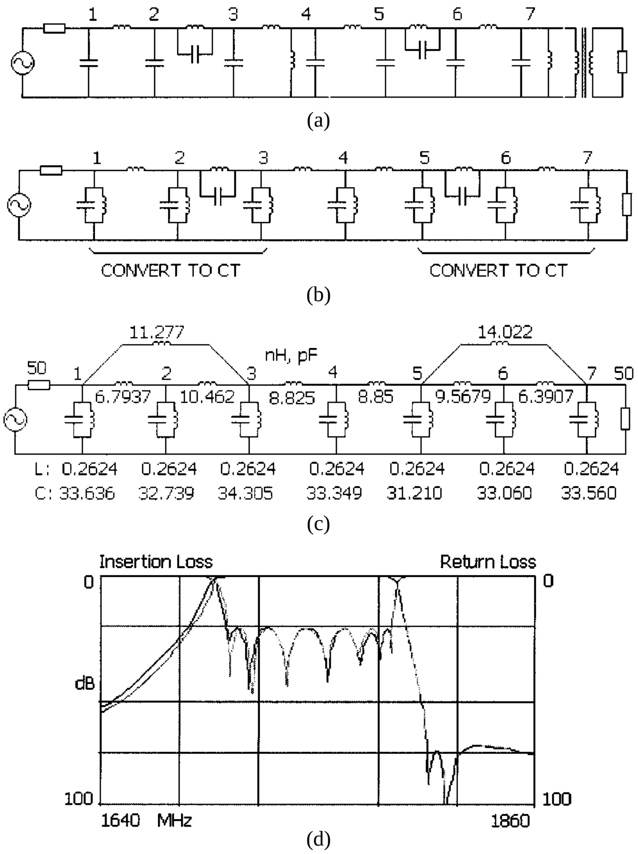
V. Practical Realization of a CT Filter

The practical value of the design procedure will be demonstrated by an example in the form of a seven-pole bandpass filter in the 1800-MHz frequency range containing two CTs. Filter specifications are as follows:

passband edges	1703.4–1787.3 MHz;
Passband return loss	>20 dB;
number of resonators	seven (degree $N = 14$);
upper stopband selectivity	>65 dB for $f \geq 1805$ MHz.

For the final realization, as an all-inductively coupled filter, the transfer function will need to have one TZ at $f = 0$. The high selectivity in the upper stopband is provided by placing two $j\omega$-axis FTZs close to the upper passband edge. With four degrees coming from the FTZs, this leaves us with nine TZs at infinity. The extracted initial network is shown in Fig. 6(a). It is then transformed into the direct-coupled shunt resonator form with two notch-type series arms creating the two desired TZs at finite frequencies, shown in Fig. 6(b). The relevant circuit sections are then converted into CT forms. The cross-couplings are represented by series inductance between nonadjacent resonators 1–3 and 5–7, as shown in Fig. 6(c). Here, the element values given can provide coupled TEM-resonator realization parameter data, namely, coupling coefficient matrix and loaded Q's. Fig. 6(d) shows the comparison between theoretical prediction and measured performance. The filter was realized in quasi-combline form. The coupling aperture dimensions were found by an iterative process using HFSS[1] electromagnetic (EM) simulation with extraction of filter parameters from S-parameter data. The parameters of an equivalent circuit for the realized filter were found by an extraction process involving circuit analysis and

[1]HFSS version 5.6, Agilent Technologies, Palo Alto, CA.



Coupling Coefficients

	Design data	Measurement extracted data	% difference
k12	0.0363	0.0352	-3.1
k23	0.0234	0.0239	2.0
k13	0.0218	0.0202	-7.1
k34	0.0278	0.0271	-2.6
k45	0.0278	0.0273	-1.7
k56	0.0256	0.0256	0.1
k67	0.0386	0.0382	-1.0
k57	0.0175	0.0181	3.3

(e)

Fig. 6. Development of the CT filter.

minimization. The resulting data is produced in Fig. 6(e). The measurement results compare well with the design data.

VI. Conclusion

The theory of cascade synthesis is revised to include CTs, CQs, and other N-tuplets as building blocks. Formulation of the transfer function is revised for the extraction of complex TZs in the forms that can readily be converted into CQ sections. Further circuit transformations are described to convert a direct-coupled resonator filter with finite or complex TZs into a cascade of canonical cross-coupled N-tuplets. Two examples are presented involving CT and CQ sections. This approach helps to transfer the wealth of knowledge on cascade synthesis of filters to a cross-coupled filters' arena. In this approach, each one of the large number of possible alternative equivalent cross-coupled solutions can be developed in a systematic and controlled manner, thus enabling designers to compare the alternatives and select or tailor the best solution. This feature may be an advantage compared to the techniques using optimization, which may overlook some of the possible alternative equivalent solutions.

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